Wave Interaction

When an electromagnetic wave (radio, light, etc) hits a surface, it can do one or a mix of six things[1]:

Here, we'll start with refraction and reflection.

Principle of Least Time

Imagine you're on the beach when you suddenly notice a child in distress in the water. You're a good swimmer but let's say you can run twice as fast as you can swim. What do you do?

Option 1: You make a B-line for the child because the shortest distance between two points is a straight line.

While it's true that this straight line is the *shortest* distance, it's not necessarily the *fastest* path. The problem here is that the water slows you down too much. It's better to cover more ground where you're faster and less where you're slower.

Option 2: You run until you're as close to the child as possible before jumping in the water to swim as little as possible.

That path might be faster than the previous one, but it's not the fastest. Here's a thought experiment:

- Imagine that you could swim as fast as you can run, then Option 1 would be the fastest path because there would be no difference between running or swimming so the shortest path would also be the quickest.
- Now imagine that you could run only *slightly* faster than you can swim. Does that mean you should go all out and run all that distance to spend as little time in the water as possible? If so, how would running yet faster change the path?

Option 2 would be the path to take if you could instantly teleport on the beach (but not in the water). In this case, you'd want to teleport as close to the child as possible, then swim the rest of the way. This path is when you can run infinitely fast.

Option 3: For regular running speeds, the quickest path is to enter the water somewhere in between.

It turns out that, people have a pretty good intuition of where that “somewhere” is. But using Calculus, it's possible to find exactly where to enter the water to get to the child as quickly as possible.

Refraction

In science classes, we learn that the speed of light is roughly 300,000,000 meters per second.[2] But that's the speed of light in empty space. In air, glass, or water, light slows down. And since light has different speeds in different media, it means that, even for light, the quickest way to get from point A to point B is **not** necessarily a straight line.

If you shine a beam of light through a piece of glass, it will bend so as to get to the other side as quickly as possible.[3]

This principle is called Fermat's Principle of Least Time and in first year Calculus, students use this principle to derive Snell's Law of Refraction, taught in high school physics, which relates the angles of incidence and refraction to the refractive indices.

Qualitatively: If light enters a medium where it travels slower, it'll bend "inward" so as to spend less time in that medium (like the picture above).

But what if light goes into a medium where it can travel faster? Then this happens:

If this last one feels weird to you, imagine this: suppose you're a turtle who can swim twice as fast as you can walk. It makes sense that you'd want to spend more time in the water and less on the beach:

To recap:

- When going from a "quick" medium to a "slow" medium, light bends away from the surface to spend less time in the slow medium.
- When going from a "slow" medium to a "quick" medium, light does the opposite and bends towards the surface.

Total Internal Reflection

This second case (going from a "slow" medium to a "fast" medium) is really interesting because at some point, the light beam bends so much that it "exits" parallel to the surface, and then reflects like a mirror:[4]

This behaviour is a bit hard to explain without going into the math, but here's an animation that allows you to explore it:

- You can move four points around to see how the refracted ray changes: "n_1", "n_2", "Laser", and "Entry point".
- Note though that this particular animation only works if the laser is below the horizontal line.

n_1 and n_2 are the Refractive Indices of the media. They are defined as the ratio of the speed of light in vacuum to the speed of light in the media $\left(n = \frac{c}{v}\right)$. For example, if n = 2, then the speed of light is twice as *slow* in the medium as it is in vacuum. The bigger n is, the slower the speed. n=1 means that the speed is the same as the speed of light in a vacuum.

A few things to try:

- Set n_1 = 1 and Set n_2 = 2 and move the Laser and the Entry Point around. These are the paths when you can run twice as fast as you can swim. Notice that if you set n_1 = 2 and n_2 = 4, or n_1 = 2.5 and n_2 = 5, it shouldn't matter. What really matters is the relative speeds between the two media.

- Now move the laser in a straight line so that the angle θ_1 doesn't change. The refracted ray shouldn't change either. So it doesn't matter how far the laser is from the surface. What matters is the angle at which the beam hits the surface.
- Now move the laser back and forth in a semi circle around the Entry Point. Although the laser is the same distance away from the Entry Point, the angle of incidence changes so the refracted ray changes.
- Now set n_1 = 1.5 and Set n_2 = 1 and play with the laser to change its angle of incidence (important). At what angle do you notice that the refracted ray goes parallel to the surface? This is called the critical angle. Passed that angle, the ray can't go through and gets reflected instead.

Example

Here's an underwater picture VE7HZF took in a lake with a waterproof camera. The camera is completely submerged under water looking up toward the surface. Above a certain angle, it's possible to see the beach, trees, and the sky. But below a certain angle, we see the reflection of his wetsuit.

Here's a sketch of the setup:

The other cool thing about that picture is that if you zoom in on the beach, you'll see the colours separate (as if through a prism). This indicates that the index of refraction, *n*, depends on the frequency.This will be important when we relate all of this back to radio waves.

Snell's Law (Optional)

Snell's law gives the relationship between the angle of incidence and refraction depending on the refraction indices:
$$ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) $$

There are four interesting cases here:

- If $n_1 < n_2$ (high speed to low speed), then the left hand side of the equation is in danger of being less than the right hand side. To maintain the equality, $\theta_1 > \theta_2$, which means that the path curves away from the surface.
- If $n_1 > n_2$ (low speed to high speed), then the right hand side of the equation is in danger of being less than the left hand side. To maintain the equality, $\theta_1 < \theta_2$, which means that the path curves away from the surface.
- If we keep increasing n_1 compared to n_2, then θ_2 can increase to the point where it's going parallel to the surface ($\theta_2 = 90^\circ$), which means that: $\frac{n_1}{n_2} \sin(\theta_1) = 1 $. At this point, we call θ_1 the critical angle.
- If we keep increasing n_1 even further, then $\frac{n_1}{n_2} \sin(\theta_1) >1 $, which means that it's

impossible for θ_2 to keep up since $\sin(\theta_2) \leq 1$. This is when Total Internal Reflection occurs, which is what we use to “bounce” radio waves off the ionosphere (more on that next).

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1) Picture from http://www.mrwaynesclass.com/lightOptics/reading/index02.html

2) It takes light roughly 8 minutes to travel from the Sun to the Earth

3) Picture modified from Wikipedia: Refractive Index

4) Picture from Wikipedia: Total Internal Reflection