

Metric Prefix

A metric prefix is a unit prefix that precedes a basic unit of measure to indicate a multiple or fraction of the unit.

In the previous section, we saw that Mhz means a million Hertz. Here's a list of the most common ones:

| Name | Symbol | Base 10 | Decimal |
|-------|--------|------------|-------------------|
| Tera | T | 10^{12} | 1 000 000 000 000 |
| Giga | G | 10^9 | 1 000 000 000 |
| Mega | M | 10^6 | 1 000 000 |
| Kilo | k | 10^3 | 1 000 |
| hecto | h | 10^2 | 100 |
| deca | da | 10^1 | 10 |
| | | 10^0 | 1 |
| deci | d | 10^{-1} | 0.1 |
| centi | c | 10^{-2} | 0.01 |
| milli | m | 10^{-3} | 0.001 |
| micro | μ | 10^{-6} | 0.000 001 |
| nano | n | 10^{-9} | 0.000 000 001 |
| pico | p | 10^{-12} | 0.000 000 000 001 |

The decibel

A decibel (dB) is a way of saying how many times bigger (or smaller) something is compared to something else. For example, let's use the faintest sound that the human ear can detect as the basis. A normal conversation would be a million times louder, and a jack hammer would be a hundred billion times louder. To represent these vast differences in numbers, we use a logarithmic scale based on powers of 10.

Here's the idea. Start with representing numbers by their exponents instead:

| Number | Base 10 | Exponent |
|-----------|---------|----------|
| 1 | 10^0 | 0 |
| 10 | 10^1 | 1 |
| 100 | 10^2 | 2 |
| 1000 | 10^3 | 3 |
| 10 000 | 10^4 | 4 |
| 100 000 | 10^5 | 5 |
| 1 000 000 | 10^6 | 6 |

For now, let's call the exponent a "bel" (not decibel yet).

- Instead of saying that a sound is **1000** times louder, we could say that it's **3 bels** louder, and
- instead of saying that it's **10,000** times louder, we could say that it's **4 bels** louder.

But what if it's 5000 times louder?

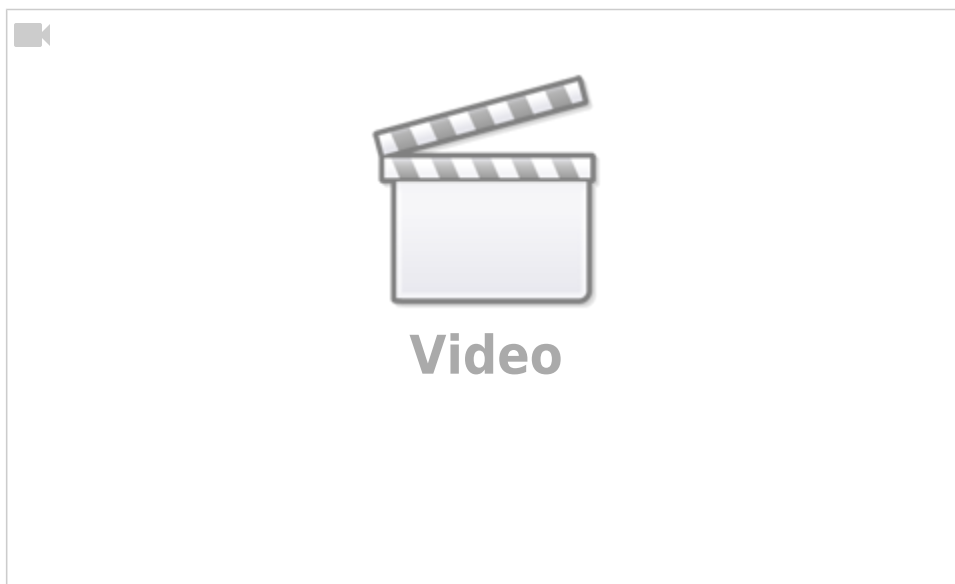
- Since 5000 is between 1000 and 10,000, it might be tempting to say that it would 3.5 bels louder (midway between 3 and 4), but let's check to see if it works: $10^{3.5} = 3162$, which is between 1000 and 10,000, but not 5000.
- By trial and error, you might get pretty close: $10^{3.7} = 5011...$ so **5000** times louder is roughly **3.7 bels**.
- But a faster way of finding the exponent of a base 10 number is to use the **Log** button on your calculator: $\log(5000) \approx 3.699$

But in the same way that *milli* means a thousandth, and *centi* means a hundredth (think millimetre and centimetre), a *deci* means a tenth. So 1 bel contains 10 decibels (like 1 metre contains 10 decimetres, 100 centimetres, or 1000 millimetres).

So another way of saying 3.7 bels is to say 37 decibels, or 37 dB. So here's our table again with a few common “in between” numbers:

| Number | Base 10 | Exponent (bels) | decibels (dB) |
|-----------|------------|-----------------|---------------|
| 1 | 10^0 | 0 | 0 |
| 2 | $10^{0.3}$ | 0.3 | 3 |
| 4 | $10^{0.6}$ | 0.6 | 6 |
| 10 | 10^1 | 1 | 10 |
| 100 | 10^2 | 2 | 20 |
| 1000 | 10^3 | 3 | 30 |
| 10 000 | 10^4 | 4 | 40 |
| 100 000 | 10^5 | 5 | 50 |
| 1 000 000 | 10^6 | 6 | 60 |

In addition to being more convenient to represent big numbers, decibels also allow us to multiply big (or small) numbers more easily by adding the dB instead. For example, take $100 \times 1000 = 100,000$. Notice how the exponents add here: $10^2 \times 10^3 = 10^5$. So 100×1000 is the same as saying $20 \text{ dB} + 30 \text{ dB} = 50 \text{ dB}$. That is, adding the dB representation of numbers, is the same as multiplying those numbers. Here's Dave explaining this in more details:



Alternative Formulation

If you like formulas, what I did above was define the dB *implicitly* as:

$$\text{ratio} = 10^{\left(\frac{\text{dB}}{10}\right)}$$

For example, a ratio of 1000 is equivalent to 30 dB because:

$$1000 = 10^{\left(\frac{30}{10}\right)}$$

But what you'll often find in books is the following *explicit* definition:

$$\text{dB} = 10 \log(\text{ratio})$$

I personally prefer the *implicit* definition because I find it easier to picture the dB in terms of powers of 10, where as I can't really picture the log function that well. But these two formulations are equivalent, so use the one you like most.²⁾

dBm

A related measurement is the dBm. While the decibel (dB) is a ratio between two quantities (saying “20 dB” is the same as saying “100 times more”), the dBm is a ratio between one quantity and 1 mW. That is, it's a measures of how much stronger (or weaker) the power of something is compared to 1 milliwatt.

For example, a typical fibre optic light signal to a house for internet has a light level of -15 dBm (or 0.03 mW)

$$-15 \text{ dBm} = 10^{-1.5} \text{ mW} \approx 0.03 \text{ mW}$$



¹⁾

Depending on your calculator, you might have to type it in this order `5|Log|`

²⁾

These two definitions are the same because the log function is the inverse of the power function. That is, it helps us find the exponent when we know the other quantities. Mathematically, it is defined such that if $a = 10^b$, then $b = \log(a)$