Metric Prefix

A metric prefix is a unit prefix that precedes a basic unit of measure to indicate a multiple or fraction of the unit.

In the previous section, we saw that Mhz means a million Hertz. Here's a list of the most common ones:

Name	Symbol	Base 10	Decimal
Tera	T	10^{12}	1 000 000 000 000
Giga	G	10^{9}	1 000 000 000
Mega	M	10^{6}	1 000 000
Kilo	k	10^{3}	1 000
hecto	h	10^{2}	100
deca	da	10^{1}	10
		10^{0}	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	µ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001

The decibel

A decibel (dB) is a way of saying how many times bigger (or smaller) something is compared to something else. For example, let's use the faintest sound that the human ear can detect as the basis. A normal conversation would be a million times louder, and a jack hammer would be a hundred billion times louder. To represent these vast differences in numbers, we use a logarithmic scale based on powers of 10.

Here's the idea. Start with representing numbers by exponents of 10:

Number	Base 10	Exponent
1 =	10^{0}	0
10 =	10^{1}	1
100 =	10^{2}	2
1000 =	10^{3}	3
10 000 =	10^{4}	4
100 000 =	10^{5}	5
1 000 000 =	10^{6}	6

If you're math is a little rusty, notice how the exponent counts the number of zeros after the one (1 000 000 has 6 zeros so it's equal to 10^{6})

For now, let's call the exponent a “bel” (not decibel yet). For example:

- Instead of saying that a sound is **1000 times** louder, we could say that it's **3 bels** louder, and
- Instead of saying that it's **10,000 times** louder, we could say that it's **4 bels** louder.

So the bel is just the number of zeros after the 1. But what if the number doesn't start with 1. For example: what if it's 5000 times louder?

- Since 5000 is between 1000 and 10,000, it might be tempting to say that it would 3.5 bels louder (midway between 3 and 4), but let's check to see if it works: $10^{3.5}$ = 3162, which is between 1000 and 10,000, but not 5000.
- By trial and error, you might get pretty close: $10^{3.7}$ = 5011... so **5000** times louder is roughly **3.7 bels**.
- But a faster way of finding the exponent of a base 10 number is to use the Log button on your calculator: \$\log(5000) \approx 3.699\$[1)]

In the same way that *milli* means a thousandth, and *centi* means a hundredth (think millimetre and centimetre), a *deci* means a tenth. So 1 bel contains 10 decibels (like 1 metre contains 10 decimetres, 100 centimetres, or 1000 millimetres).

So another way of saying 3.7 bels is to say 37 decibels, or 37 dB. So here's our table again with a few common “in between” numbers:

Number	Base 10	Exponent (bels)	decibels (dB)
1 =	10^0	0	0
2 ≈	$10^{0.3}$	0.3	3
4 ≈	$10^{0.6}$	0.6	6
8 ≈	$10^{0.9}$	0.9	9
10 =	10^1	1	10
100 =	10^2	2	20
1000 =	10^3	3	30
10 000 =	10^4	4	40
100 000 =	10^5	5	50
1 000 000 =	10^6	6	60

In addition to being more convenient to represent big numbers, decibels also allow us to multiply big (or small) numbers more easily by adding the dB instead. For example, look at:

- 100 x 1000 = 100,000

Let's write the same thing again in powers of 10:

- $10^2 \times 10^3 = 10^5$

Notice that while the numbers are multiplying (100 x 1000), the exponents are adding (2 + 3). So 100 x 1000 is the same as saying 20 dB + 30 dB = 50 dB. That is, adding the dB representation of numbers, is the same as multiplying those numbers.

Look back at the previous table; if you double the power, three times in a row:

- The final power is 2 x 2 x 2 = 8 times more powerful than the original.
- In terms of dB, we can instead add 3dB three times: 3dB + 3dB + 3dB = 9dB.

Here's a quick exercise: Using only the table above, what is the dB representation of 4000?

A 4000 fold increase is the same as a 1000 fold followed by two doubling. That is:

- 1000 x 2 x 2 = 4000

Which means:

- 30dB + 3dB + 3dB = 36dB

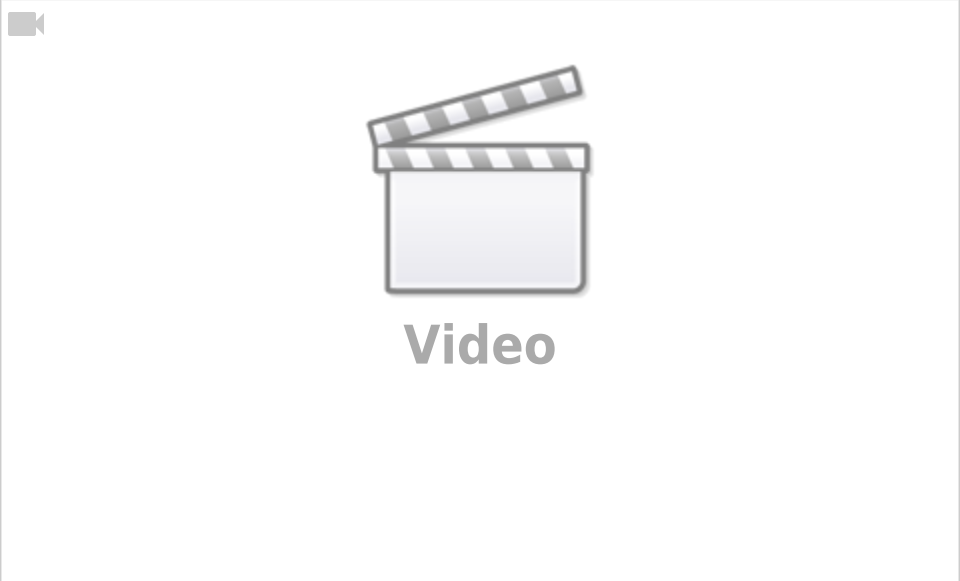
Quick calculator check: $10^{3.6} = 3981 \approx 4000$

So the only thing to memorize with decibels is that:

- a bel (10 dB) is the number of zeros after the 1. So that takes care of knowing 10dB, 20dB, 30dB, ...
- 3dB is one doubling.

With these two things, we can now estimate a bunch of in between numbers like 36dB (30dB + 3dB + 3dB), or 13dB (10dB + 3dB)

Here's Dave explaining this in more details:



Alternative Formulation

If you like formulas, what I did above was define the dB *implicitly* as:

```
\$$ \text{ratio} =
10^{\left(\frac{\text{dB}}{10}
\right)} \$$
```

For example, a ratio of 1000 is equivalent to 30 dB because:

\$$ 1000 = 10^{\left(\frac{\text{30}}{10} \right)} \$$

But what you'll often find in books is the following *explicit* definition:

```
\$$ \text{dB} =
10\log\left(\text{ratio}\right) \$$
```

While the two are definitions are mathematically equivalent[2)], I personally prefer the *implicit* definition because I find it easier to picture the dB in terms of powers of 10, where as I can't really picture the log function that well. But they are equivalent, so use the one you like most.

dBm

A related measurement is the dBm. While the decibel (dB) is a ratio between two quantities (saying “20 dB” is the same as saying “100 times more”), the dBm is a ratio between one quantity and 1 mW. That is, it's a measures of how much stronger (or weaker) the power of something is compared to 1 milliwatt.

For example, a typical fibre optic light signal to a house for internet has a light level of -16 dBm. This means the light has an output power of 25 µW (microwatts) because:

$$-16 \text{ dBm} = 1 \text{ mW} \div 10^{1.6} \approx 1 \text{ mW} \div 40 = 0.025 \text{ mW} = 25~\mu \text{W} \$$

Notice how the minus sign in front of the dB means that the power is less (not more) so it's divides (not multiply).

1)
Depending on your calculator, you might have to type it in this order 5 Log

2)
These two definitions are equivalent because the log function is the inverse of the power function. That is, it helps us find the exponent when we know the other quantities. Mathematically, it is defined such that if $a = 10^b$, then $b = \log(a) $