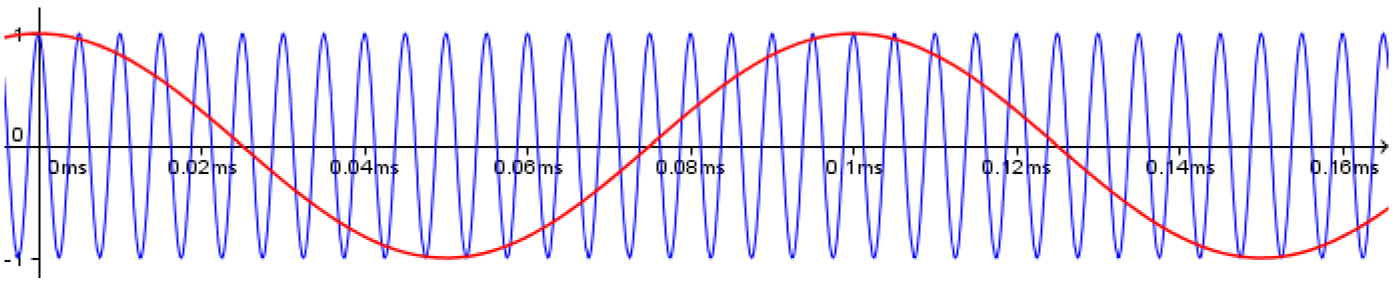
More Details: AM / FM

Here are a few more details about the AM, SSB, and FM modulation schemes introduced on the Wave Modulation page. I made the interactive animations for this section using Geogebra 4.2 (later versions can't export as html5).

For both AM and FM examples, we'll Let:

- <latex> $c(t) = \cos(2 \pi f_c t)$ </latex> be the radio carrier with frequency <latex>f_c</latex>
- <latex> $s(t) = \cos(2 \pi f_s t)$ </latex> be the baseband audio signal with frequency <latex>f_s</latex>



With the radio carrier frequency several times greater than the baseband audio signal.

AM

The resulting *Amplitude Modulated* radio wave is the **product** of the vertically shifted baseband signal and the radio carrier, which is also equal to the **sum** of the carrier and the two side bands:

<latex> \begin{align} \Big(s(t)+1\Big) \times c(t) &= \Big(\cos(2 \pi f_s t) + 1\Big) \times \cos(2 \pi f_c t) \\ &= \cos(2 \pi f_s t)\cos(2 \pi f_c t) + \cos(2 \pi f_c t)\\ &= \underbrace{\frac{1}{2} \cos\Big(2 \pi (f_c + f_s) t\Big)}_\text{USB} + \underbrace{\frac{1}{2} \cos\Big(2 \pi (f_c - f_s) t\Big)}_\text{LSB} + \underbrace{\cos(2 \pi f_c t)}_\text{Carrier} \end{align} </latex>

In line 1, I distributed the bracket, which, in line 2, gave us the carrier (last term) and a product (first term). To expend this product into the sum of the two side bands (line 3), I added these two trig identities together:

<latex> \begin{align*} \cos(A+B) =& \cos(A)\cos(B) - \sin(A)\sin(B)
\cos(A-B) =& \cos(A)\cos(B) + \sin(A)\sin(B)
\end{align} </latex>

Which gives:

<latex> \begin{align*} &\cos(A+B) + \cos(A-B) = 2 \cos(A)\cos(B)
\Rightarrow &\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B) \end{align} </latex>

Use this animation to see what happens when you vary the individual frequencies. You can use the check boxes to show or hide different waves.

Some things to try:

- Set f_s at 10 and f_c at 200 and check only the transmitted signal. You can easily imagine what the envelope (the baseband signal) should be that produced that signal. But...
- Decrease f_c slowly. At some point (around 20 or 30) the baseband signal becomes unrecoverable. This illustrates the point that to transmit a high frequency baseband, a higher frequency carrier is needed (at least 3 to 4 times the frequency of the baseband signal. This is why with digital signals, the higher the transfer speed, the higher the carrier frequency needs to be.

FM

Mathematically, FM is less intuitive and more complicated than AM to understand. The first step is to modulate the frequency by adding a scaled baseband function to it:

<latex> $$2\pi f_c \quad \rightarrow \quad 2\pi f_c + 2\pi k s(t)$$ </latex>.

- Here, f_c is the frequency of the carrier, which is a constant (this is important),
- and k is a scaling factor we can use to decide how much of a variation we allow the baseband signal to impart on the carrier frequency. When $k = 0$, there is no modulation, and the greater k becomes, the bigger the effect is.

Now, it might be tempting to simply substitute this sum in the wave like so:

<latex> $$ \cos(2\pi f_c t) \quad \rightarrow \quad \cos\Big(\big(2\pi f_c + 2\pi k s(t)\big) t\Big) $$ </latex>

but that's not quite right because the frequency is derived from the change in angle. To solve this properly, we need some calculus and deduce the angle from our new frequency:

<latex> $$ \frac{d}{dt}\theta(t) = 2\pi f_c + 2\pi k s(t) \qquad \Rightarrow \qquad \theta(t) = 2\pi f_c t + 2\pi k \int_0^{t}s(\tau) d\tau $$ </latex>

The frequency modulated transmission is actually given by:

<latex> $$ \cos\Big(2\pi f_c t + 2\pi k \int_0^{t}s(\tau) d\tau\Big) $$ </latex>

In our particular example, with <latex> $s(t) = \cos(2 \pi f_s t)$ </latex>, the modulated radio signal becomes:

<latex> $$\begin{align} \cos\Big(2 \pi f_c t + 2\pi k \int_0^{t}s(\tau)d\tau\Big) &= \cos\Big(2 \pi f_c t + 2\pi k \int_0^{t}\cos(2 \pi f_s \tau)d\tau\Big) \\ &= \cos\Big(2 \pi f_c t + k \sin(2 \pi f_s t)\Big) \end{align}$$ </latex>

For more details about FM, see: http://www.ece.umd.edu/~tretter/commlab/c6713slides/ch8.pdf

Use this animation to see what happens when you vary the individual frequencies. You can use the check boxes to show or hide different waves.

Some things to try:

- Set f_s at 10 and f_c at 200 and check only the transmitted signal. Notice how when the baseband is high, the **transmitted wave** is “tight” (ie, its frequency is high), and vise-versa. But...

- Decrease f_c slowly. At some point (around 20 or 30) that pattern becomes unnoticeable. Again, this illustrates the point that to transmit a high frequency baseband, a higher frequency carrier is needed (at least 3 to 4 times the frequency of the baseband signal. This is why with digital signals, the higher the transfer speed, the higher the carrier frequency needs to be.
- Increase and decrease **k** to see the effect it has on the transmitted wave. The greater **k**, the more bandwidth the resulting signal uses. This dictates the difference between “Narrow Band FM” and “Wide Band FM”.

See https://electronicspost.com/narrow-band-fm-wide-band-fm/

PM

Phase Modulation is not usually discussed in ham radio courses, but after understanding FM, we pretty much get PM for free... Recall that for the wave $\cos(2\pi f + \phi)$, f is the frequency and ϕ is the phase shift. For a pure tone, both of these are constant.

- With FM, we saw that modulating the frequency led to $\cos\Big(2 \pi f_c t + 2\pi k \int_0^{t}s(\tau)d\tau\Big)$.
- With PM, it leads to $\cos\Big(2 \pi f_c t + 2\pi k s(t)\Big)$

Essentially, with PM, we simply let ϕ vary with the baseband $s(t)$. But the thing to notice is that PM looks a lot like FM. In fact, an FM signal modulated by $s(t)$ is the same as a PM signal modulated by $\int_0^{t}s(\tau)d\tau$. In other words, the receiver needs to know if the signal was modulated in FM or PM since both wave forms look similar.