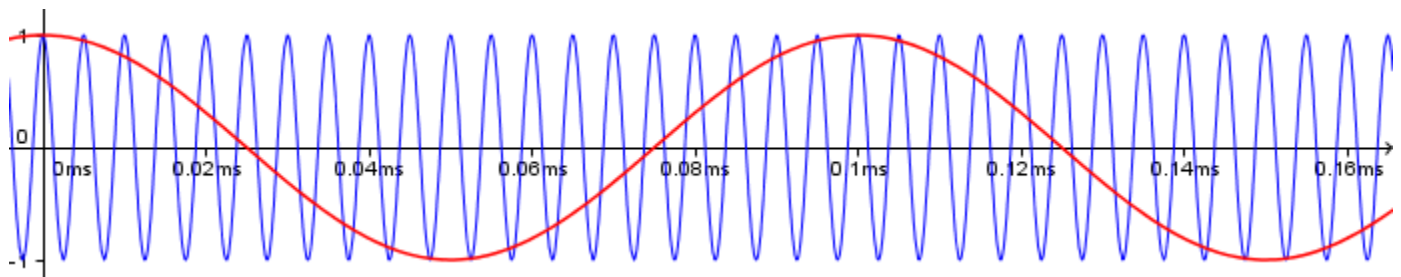


## More Details: AM / FM

Here are a few more details about the AM, SSB, and FM modulation schemes introduced on the [Wave Modulation](#) page. I made the interactive animations for this section using [Geogebra 4.2](#) (later versions can't export as html5).

For both AM and FM examples, we'll Let:

- $c(t) = \cos(2\pi f_c t)$  be the **radio carrier** with frequency  $f_c$
- $s(t) = \cos(2\pi f_s t)$  be the **baseband audio signal** with frequency  $f_s$



With the radio carrier frequency several times greater than the baseband audio signal.

## AM

The resulting *Amplitude Modulated* radio wave is the **product** of the vertically shifted baseband signal and the radio carrier, which is also equal to the **sum** of the carrier and the two side bands:

$$\begin{aligned} \Big(s(t)+1\Big) \times c(t) &= \Big(\cos(2\pi f_s t) + 1\Big) \times \cos(2\pi f_c t) \\ &= \underbrace{\cos(2\pi f_s t)\cos(2\pi f_c t)}_{\text{USB}} + \underbrace{\cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\cos(2\pi (f_c - f_s) t)}_{\text{LSB}} \end{aligned}$$

In line 1, I distributed the bracket, which, in line 2, gave us the carrier (last term) and a product (first term). To expand this product into the sum of the two side bands (line 3), I added these two trig identities together:

$$\begin{aligned} \cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \\ \cos(A-B) &= \cos(A)\cos(B) + \sin(A)\sin(B) \end{aligned}$$

Which gives:

$$\begin{aligned} \cos(A+B) + \cos(A-B) &= 2\cos(A)\cos(B) \\ \Rightarrow \cos(A)\cos(B) &= \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B) \end{aligned}$$

Use this animation to see what happens when you vary the individual frequencies. You can use the check boxes to show or hide different waves.

Some things to try:

- Set  $f_s$  at 10 and  $f_c$  at 200 and check only the transmitted signal. You can easily imagine what the envelope (the baseband signal) should be that produced that signal. But...
- Decrease  $f_c$  slowly. At some point (around 20 or 30) the baseband signal becomes unrecoverable. This illustrates the point that to transmit a high frequency baseband, a higher frequency carrier is needed (at least 3 to 4 times the frequency of the baseband signal. This is why with digital signals, the higher the transfer speed, the higher the carrier frequency needs to be.

## FM

Mathematically, FM is less intuitive and more complicated than AM to understand. The first step is to modulate the frequency by adding a scaled baseband function to it:

$$\omega_c \rightarrow \omega_c + 2\pi k s(t)$$

- Here,  $f_c$  is the frequency of the carrier, which is a constant (this is important),
- and  $k$  is a scaling factor we can use to decide how much of a variation we allow the baseband signal to impart on the carrier frequency. When  $k = 0$ , there is no modulation, and the greater  $k$  becomes, the bigger the effect is.

Now, it might be tempting to simply substitute this sum in the wave like so:

$$\cos(\omega_c t) \rightarrow \cos(\omega_c + 2\pi k s(t) t)$$

but that's **not quite right** because the frequency is derived from the change in angle. To solve this properly, we need some calculus and deduce the angle from our new frequency:

$$\frac{d}{dt} \theta(t) = \omega_c + 2\pi k s(t) \rightarrow \theta(t) = \omega_c t + 2\pi k \int_0^t s(\tau) d\tau$$

The frequency modulated transmission is actually given by:

$$\cos(\omega_c t + 2\pi k \int_0^t s(\tau) d\tau)$$

In our particular example, with  $s(t) = \cos(2\pi f_s t)$ , the modulated radio signal becomes:

$$\cos(\omega_c t + 2\pi k \int_0^t s(\tau) d\tau) = \cos(\omega_c t + 2\pi k \int_0^t \cos(2\pi f_s \tau) d\tau) = \cos(\omega_c t + k \sin(2\pi f_s t))$$

For more details about FM, see: <http://www.ece.umd.edu/~tretter/commlab/c6713slides/ch8.pdf>

Use this animation to see what happens when you vary the individual frequencies. You can use the check boxes to show or hide different waves.

Some things to try:

- Set  $f_s$  at 10 and  $f_c$  at 200 and check only the transmitted signal. Notice how when the **baseband** is high, the **transmitted wave** is “tight” (ie, its frequency is high), and vice-versa. But...

- Decrease  $f_c$  slowly. At some point (around 20 or 30) that pattern becomes unnoticeable. Again, this illustrates the point that to transmit a high frequency baseband, a higher frequency carrier is needed (at least 3 to 4 times the frequency of the baseband signal. This is why with digital signals, the higher the transfer speed, the higher the carrier frequency needs to be.
- Increase and decrease  $k$  to see the effect it has on the transmitted wave. The greater  $k$ , the more bandwidth the resulting signal uses. This dictates the difference between “Narrow Band FM” and “Wide Band FM”.

See <https://electronicspost.com/narrow-band-fm-wide-band-fm/>

## PM

*Phase Modulation* is not usually discussed in ham radio courses, but after understanding FM, we pretty much get PM for free... Recall that for the wave  $\cos(2\pi f t + \phi)$ ,  $f$  is the frequency and  $\phi$  is the phase shift. For a pure tone, both of these are constant.

- With FM, we saw that modulating the frequency led to  $\cos\left(2\pi f_c t + 2\pi k \int_0^t s(\tau) d\tau\right)$ .
- With PM, it leads to  $\cos\left(2\pi f_c t + 2\pi k s(t)\right)$

Essentially, with PM, we simply let  $\phi$  vary with the baseband  $s(t)$ . But the thing to notice is that PM looks a lot like FM. In fact, an FM signal modulated by  $s(t)$  is the same as a PM signal modulated by  $\int_0^t s(\tau) d\tau$ . In other words, the receiver needs to know if the signal was modulated in FM or PM since both wave forms look similar.

